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Value for Money of Retirement Insurance Plans in Malaysia with Consideration of Longevity Factor

Nurin Haniah Asmuni[1,2]*, Sharifah Nazatul Shima Syed Mohamed Shahruddin[1] and Norkhairunnisa Redzwan[1]

[1]Centre of Actuarial Studies, Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia
[2]Department of Statistics and Actuarial Science, Faculty of Mathematics, University of Waterloo, Ontario, N2L 3G1, Canada

ABSTRACT

The rising cost of pensions as a result of longevity risk is an emerging global issue. In Malaysia, the mortality rate has gradually improved over time and consequently, the old-age dependency ratio has also increased. Thus, there is a need to further develop voluntary retirement schemes, such as annuities, in Malaysia to help retirees sustain their retirement income. However, the Malaysian private pension market is very small and there is a lack of understanding of the products among retirees. This study aims to calculate the value for money of retirement insurance products in Malaysia based on age and gender. The value for money calculation provides financial information to assist customers in selecting their optimal plan upon retirement. The value for money calculation was performed using the money's worth ratio (MWR) approach. Mortality rates are projected using the Lee-Carter model to account for longevity risk. The findings comprise the MWR values calculated for two private retirement products available in the Malaysian market, where one features an investment-linked component and the other is a deferred annuity. Our findings show that the plain deferred annuity gives a significantly higher

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E-mail addresses:
nurin@fskm.uitm.edu.my (Nurin Haniah Asmuni)
snshima@fskm.uitm.edu.my (Sharifah Nazatul Shima Syed Mohamed Shahruddin)
khairunnisa@fskm.uitm.edu.my (Norkhairunnisa Redzwan)
*Corresponding author

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value for money than the investment-linked product for all ages and both genders.

Keywords: Lee-Carter model, longevity, money's worth ratio, mortality, retirement

INTRODUCTION

Population ageing is a demographic change phenomenon that affects future retirement costs as retirees are expected to spend increasing amounts of time in retirement. The longevity risk associated with the improvement of mortality rates everywhere in the world is a global issue causing uncertainty in future retirement costs. Asher and Bali (2015) summarised the multi-pillar retirement system in Malaysia as follows: i) defined benefit plans – the government pension scheme, ii) defined contribution plans – the Employees' Provident Fund (EPF), and iii) voluntary schemes – annuities, private pension, and retirement insurance plans. They underscored the role of the financing mix in managing retirement spending. Owing to the huge increase in the projected old-age dependency ratio from 6% in 1990 to 13% in 2021, pension costs have continued to increase in recent years, creating a financial burden for public pension fund providers (Park & Shin, 2011). Policymakers are looking for better ways to sustain pension funds and a financing mix comprising defined benefit and defined contribution plans appear promising for the future. On top of that, retirees' awareness of private retirement insurance plans is also crucial in providing sufficient retirement income to avoid retirees outliving their assets.

Currently, in the decumulation phase, the EPF allows retirees to choose how to manage their retirement savings without any restriction on how they convert the fund to a stream of income during retirement. The EPF allows lump-sum withdrawal upon retirement as well as offering several withdrawal schemes for health and education purposes before retirement. Therefore, it is important for EPF members to effectively manage their retirement savings by considering all available retirement-linked instruments in the market. For instance, the purchase of annuities or any retirement-linked insurance product is currently completely voluntary and based on the retiree's decision. Low financial literacy among retirees makes the development of voluntary retirement schemes challenging. Alexandrova and Gatzert (2019) highlighted this low financial literacy as one of the reasons for the low annuity demand in the market. Thus, understanding the value for money of such products is critical in educating future retirees for optimally managing their retirement funds.

Analysis of the value for money of retirement insurance products such as annuities has long been discussed in the literature, mainly within the context of developed countries. The concept of the money's worth ratio (MWR) was first introduced by Mitchell et al. (1999) for evaluating the value for money of life annuities available in the US private market in 1995. According to Milevsky (2013), at least three dozen research articles published in the past two decades have examined the MWR of annuities in countries ranging from Singapore to Chile. A recent study by Aquilina et al. (2017) analysed the MWR of life annuities in the UK following the ban on gender-based price discrimination in the UK.

In the Asian region, our literature search found only two countries for which MWR results have been published namely, Singapore and Malaysia (Asmuni & Purcal, 2018; Fong et al., 2014). This is because the retirement insurance plan market is immature in this region, where most countries are categorized as developing. For developing economies in the region such as Malaysia, the market for innovative retirement products to help mitigate longevity risks, such as annuities and longevity bonds, is very small. Asmuni and Purcal (2018) reported on the history of the annuity market in Malaysia, which was suspended in 2001 because of mis-selling issues and a lack of understanding of the product. Subsequently, annuities only returned to the market a decade after their suspension.

Nevertheless, the demand for annuities has traditionally been very low in Malaysia. The penetration rate for annuities in Malaysia was close to zero over the past decade. Annuities represent only 0.75% of the total number of policies in force for direct insurers in 2018 (Central Bank of Malaysia, 2019). In contrast, the take-up rates in Chile and Switzerland are high with a significantly growing number of policies sold over time. In Chile, almost two-thirds of all retirees do annuities whilst in Switzerland, only around 10 to 30 percent of all individuals cash out their pension (Bütler & Staubli, 2010; Rocha & Thorburn, 2007). Thus, further research is urgently needed to support the development of voluntary retirement schemes in this country.

The aim of this work is to build on prior research by calculating the value for money of the retirement insurance products that were available on the private Malaysian market in 2018 based on age and gender. The longevity factor is incorporated in the analysis by taking into account the improvement in the mortality rate over time using the Lee-Carter model. This is the first study to evaluate the value for money of retirement insurance products where one features an investment-linked component (Product A) and the other is a deferred annuity plan (Product B). We consider these two products owing to their distinct features, where Product A provides a yearly retirement income based on the retiree's investment allocation decision. In particular, a retiree may choose to invest some proportion of their pension fund into a high-risk investment fund over the insurance term. Thus, the expected return of Product A depends on this investment decision. On the other hand, Product B provides a fixed yearly retirement income determined at the time of purchase with no option for retirees to choose their investment allocation. This value for money calculation and comparison will contribute to improving the financial literacy of retirees, thus helping them in selecting the optimal plan upon retirement.

METHOD

The MWR approach was applied to evaluate the worthiness of a retirement insurance purchase. This concept has been used widely in economics to evaluate the value for money of annuity products (Brown, 2003; Fong, 2011; Ganegoda & Bateman, 2008). Intuitively, the MWR formula calculates the value of a benefit received as a ratio of a dollar premium paid for the insurance product. The MWR formula used in the analysis is outlined below:

$$MWR = \frac{EPDV}{P} \qquad [1]$$

where $EPDV$ is the expected present discounted value of all benefits covered by the product and P is the premium paid for the product. $EPDV$ is calculated as follows:

$$EPDV = \sum_{t=0}^{y-x} v^t \; {}_t p_x^{\overline{11}} A_t + v^t \; {}_{t-1}p_x^{\overline{11}} \; {}_1p_{x+t-1}^{12} B_t + v^t \; {}_{t-1}p_x^{\overline{11}} \; {}_1p_{x+t-1}^{13} B_t. \qquad [2]$$

The ${}_1p_x^{12}$ value is the transition rate from State 1 (healthy) to State 2 (total permanent disabled, TPD), whereas the ${}_1p_x^{\overline{11}}$ is the probability of retirees staying in State 1 (healthy) in a year. In the event of survival, the annuity stream of income A_t will be paid. On the other hand, if TPD (State 2) or death (State 3) occurs, the insured will receive a lump-sum benefit payment of B_t. The age at purchase is represented by x whilst y represents the age at which the last payment is made. Equation [2] is applied separately for males and females for ages at purchase of 30, 40, and 50 years.

The mortality rate is estimated based on population mortality data from 2000 to 2016 obtained from the World Health Organization (WHO) and the total permanent disability rate is estimated from data in the World Report on Disability (WHO, 2011). Owing to the limited amount of publicly available data, we assume a stationary rate for the TPD transition rate. A time-inhomogeneous multiple states Markov model is applied to estimate these rates (Haberman et al., 1997; Rickayzen & Walsh, 2002). We follow the conversion method reported by Gatenby (1991) to estimate the TPD state transition rate from prevalence data. Figure 1 illustrates the Markov model developed in our analysis.

Figure 1. Markov model for retirement insurance products

Retirement Insurance Products

A distinct element that differentiates Product A from Product B is that Product A comprises an investment-linked component where some proportion of the premium paid will be invested in a high-risk investment fund. Though such products offer the opportunity for higher returns, the variability is also high owing to exposure to investment risk. Thus, the income payment stream depends on the performance of the investment fund. Furthermore, the product also offers a lump-sum payment for TPD or a death benefit. On the contrary, Product B is a deferred annuity product that pays a stream of annuity income without investment options. TPD or death benefits are also offered. The premium and benefit payment illustrations for both products are presented in Appendix 1 and 2. For Product A, the projected investment returns of X% and Y% are provided by the insurance company together with the projected benefit associated with each investment return. In this sales quotation, the company assumes that 25% of the premium paid is allocated to an equity fund (high risk) whilst the remaining 75% is allocated to a bond fund (low risk). For Product B, the benefit illustration is fixed with no choice of investment offered to the customer.

Mortality Factors

The mortality rates are projected using the well-known Lee-Carter mortality projection model, as shown in equation [3] (Lee & Carter, 1992). Since the data obtained from the WHO is in age group categories, the Heligman-Pollard formula is applied to convert the age group mortality to a single age mortality rate (Heligman & Pollard, 1980; Ibrahim & Siri, 2011; Kostaki, 1991).

$$ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \qquad [3]$$

where $m_{x,t}$ is the central death rate of age x in year t, and the vectors a, b, k are estimated by finding the least square solution using the singular value decomposition (SVD) method. The R statistical software codes used to solve the vectors can be found in Appendix 3. A forecast package is applied in our coding to project the mortality rates (Hyndman et al., 2019).

RESULTS

The fitted mortality rates for the Malaysian male and female populations are shown in Figure 2 and Figure 3, respectively. The sum of squared error obtained from the fitting procedure using the Lee-Carter model is close to zero for all ages and both gender categories, which indicates that the model fits well with the Malaysian mortality rate data.

These rates are then projected to the year 2088 to match our assumption of the maximum life span of 100 years old. If a customer purchases a retirement insurance plan at the age of 30 in 2018, the consumer will reach the age of 100 in 2088.

The MWR values are calculated for Products A and B for purchase ages of 30, 40, and 50 years for each gender category. For Product A, the investment-linked part of the product allows the customer to choose the proportion of the premium to be allocated to high-risk investments, such as equity. Thus, the investment returns vary depending on the investment performance of the selected portfolio. In this paper, the calculations are performed based on the projected returns provided by the company arising from favourable (X% annually) and less favourable market conditions (Y% annually). On the other hand, the assumption of interest rates used for the valuation of Product B valuation is based on the risk-free rate of return obtained from the Malaysia Government Securities' return. The MWR values for Products A and B are shown in Table 1 and Table 2, respectively.

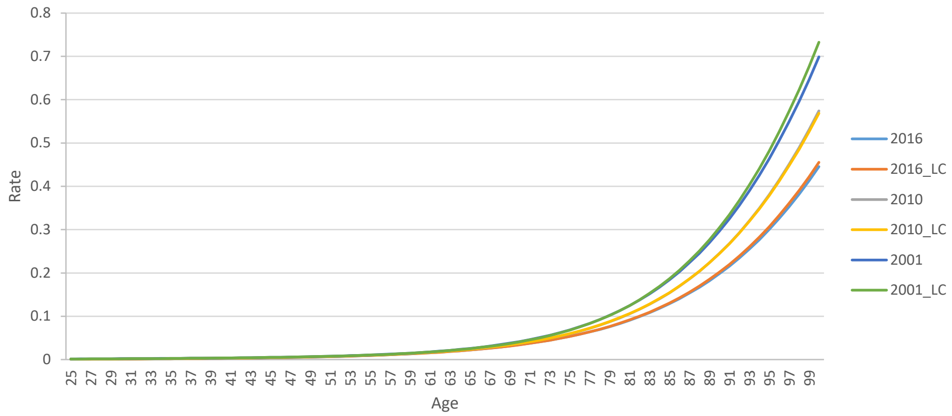

Figure 2. Lee-Carter fitted mortality rates for males[1]

1 The empirical mortality rate data is labelled using the base year of the observed mortality rate whereas the fitted mortality rate obtained from the Lee-Carter model is labelled as 'year_LC'.

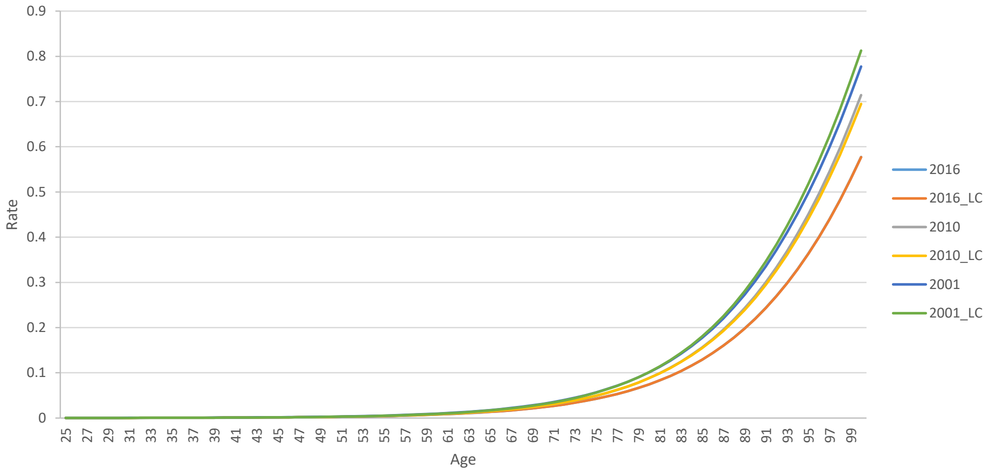

Figure 3. Lee-Carter fitted mortality rates for females[1]

Table 1
The money's worth ratio (MWR) values of Product A

Age	Male		Female	
	X%	Y%	X%	Y%
30	0.4671	0.7212	0.4561	0.7157
40	0.4878	0.7313	0.4670	0.7210
50	0.5138	0.7429	0.4772	0.7231

Table 2
The money's worth ratio (MWR) values of Product B

Age	Male	Female
30	1.0123	1.0383
40	1.0192	1.0365
50	0.9628	0.9708

DISCUSSION

The MWR value represents the expected benefit received per dollar of premium paid for the insurance product. A value of greater than 1 means that the expected benefit is higher than the premium, indicating that the product offers very good value for the money. However, in the insurance market, most private or profit-based companies offer products with premium charges called loading fee to cover administrative costs, commission, and profit allocation. Milevsky (2013) summarised the MWR values of annuities calculated in developed countries including the US, the UK, Canada, and Australia. All of these countries have MWR values below 1, ranging from 0.814 to 0.965 and 0.852 to 0.937 for males and females, respectively. In addition, a study by Fong et al. (2014) showed that the MWR value for a life annuity purchased from a commercial insurer in Singapore was on average 0.948 and 0.957 for male and female categories respectively in 2007. Thus, it is not unusual to have an MWR value of less than 1. A lower MWR value indicates that a higher loading fee needs to be paid to receive the expected benefit from the product.

Based on Table 1, given an expected return of X% where the annual return is on average 7.4%, assuming favourable market conditions, the value for money of Product A is very low for all ages for both male and female categories. Thus, consumers would be better off investing their premium in the market without purchasing the product. On the other hand, the value for money is about 30% higher if the expected return of Y% is applied. The rate of return Y% is expected to be 3.5% on average, close to Malaysia's average annual risk-free rate of return. The MWR value increases with age, showing no incentive for purchasing the product at an early age. Since Product A is an investment-linked retirement product, the MWR value ranges from 0.46 to 0.74 depending on the expected return.

In contrast, the value for money of Product B is about 5% and 7% higher for males and females, respectively, for the purchase age 30 in comparison to purchase age 50, showing an incentive for purchasing at an earlier age. MWR values higher than one for ages 30 and 40 for both genders indicates that the product offers very good value for money to consumers. The MWR value for the age 50 category is also good since it is only slightly less than 1 and comparable to other countries' results. Overall, Product B has a higher value for money for all categories compared to Product A based on the MWR approach.

Let us now focus on the MWR results for both products according to gender. For Product A, the MWR value is slightly lower for females. Conversely, for Product B, the MWR value is slightly higher for females. This finding is supported by the projected benefits of both products. For the same premium, product B offers slightly different TPD and death benefits for females as compared to males. In our calculation, this resulted in a higher EPDV of benefit value (as calculated using equation [2]) for the female category, whilst Product A offers exactly the same benefit regardless of gender. Our results indicate that the equalised benefit provision favours insured males owing to the higher mortality rates for males as compared to females. The projected mortality rates for males are higher in comparison to females, especially with increasing age, as shown in Figure 2 and Figure 3. Thus, males are more likely to receive the TPD, or death benefit is higher. If equalised benefit and premium charges are imposed for both genders, this will become favourable for prospective male buyers.

Overall, our findings provide two important lessons for potential insurance buyers. First, the investment-linked product considered in this paper, which we called Product A, offers benefits that are heavily weighted on the TPD and death benefit. The projected yearly income of RM3,500 is rather small in comparison to the yearly income offered under Product B (refer to Appendix 1 and 2). It is well explained in the literature that the purpose of purchasing a retirement insurance product is to counteract improving mortality rates or the so-called longevity risk for retirees (Alexandrova & Gatzert, 2019; Asmuni & Purcal, 2018; Fong et al., 2014; Milevsky, 2013). Thus, such retirement products should offer benefits that are heavily weighted towards yearly income since the likelihood of receiving this income increases with increasing longevity. This is also an important input for actuaries working on retirement product design for insurance companies.

Second, the low value for money of Product A can also be explained by the high loading fees associated with the product (which include the yearly fund management charges[2]). These kinds of expenses are not payable under the deferred annuity product. Thus, even though some consumers prefer to have flexibility in choosing their investment funds, they have to be aware of the charges associated with this option.

CONCLUSION

Population ageing has become an indisputable issue over the past decade since mortality rates have improved quite substantially almost everywhere. The expected increase in years spent in retirement requires proper financial knowledge to optimally plan for a sustainable retirement income. To date, the market for innovative products offering retirement benefits to mitigate the longevity risk is still small in Malaysia. As the population ageing issue becomes more prevalent, the demand for innovative retirement products is also expected to increase. Future retirees should be equipped with appropriate financial knowledge on such products to make good decisions for their retirement planning.

In developed countries, the study of the value for money of retirement insurance plans such as annuities has been a central focus in the literature. However, owing to the limited market for such products in Malaysia, few studies have been carried out in this country. Thus, this study aimed to analyse the value for money of the retirement insurance products available on the private market in Malaysia in 2018 using the MWR approach. Mortality rates were projected using the Lee-Carter model to incorporate longevity risk in the analysis.

Based on the products currently available in the Malaysian market, we found a significant difference in the MWR values of the two retirement insurance products, one featuring an investment-linked component and the other being a deferred annuity. In conclusion, the deferred annuity offers higher value for money for retirees of all ages in both male and female categories. Hence, it provides a reliable and good-value retirement income for retirees. The retirement insurance plan with an investment-linked component has low value for money owing to two factors, the heavily weighted TPD and death benefit as compared to the yearly retirement income and the higher loading fees, which include the fund management charges.

Our results offer some important insights for potential insurance buyers and providers. A retirement insurance plan should concentrate on providing the highest possible yearly retirement income (based on the premium paid) in order to cope with improving mortality rates. This will offer a retirement plan with good value for money that is worth purchasing. In addition, potential buyers should be aware of the additional charges that come in exchange for the flexible choice of investments offered under a retirement product with an investment-linked component. Even though there is the potential for higher

2 According to the sales illustration provided by the insurance company, the fund management charge is 1.5% (as a percentage of invested funds) for the equity fund and 1.0% for the bond fund.

returns associated with risky investments, these returns are not guaranteed and the benefits received will depend entirely on the investment fund's performance. Our results are limited to the analysis performed using the projected returns and benefits provided by the insurance company in the sales illustration. The company assumes that 25% of the premium paid is allocated to high-risk investments (equity funds) whilst the remaining 75% is allocated to low-risk investments (bond funds).

The analysis in this paper only considered retirement insurance plans that are currently available on the private market in Malaysia. There is a huge gap in the voluntary retirement schemes market in Malaysia in comparison to other developed countries. In particular, the products currently offered by insurers only provide a yearly retirement income for a limited-term, such as 10 years. Although such products offer additional income protection upon retirement; it does not solve the major issue of longevity risk for retirees. The uncertainty regarding the amount of time spent in retirement may leave retirees with no income at all if they live longer than expected. Clearly, there is room for further development of voluntary retirement schemes, such as nominal life annuities or inflation-indexed life annuities that pay a yearly retirement income for the duration of the retiree's life.

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APPENDICES

Appendix 1

Illustration of benefit payment for males[3]

Age	Product A			Product B	
	Annuity Income	TPD/Death (X%)	TPD/Death (Y%)	Annuity Income	TPD/Death
30	3500	61091	41500	14762	53709
40	3500	61091	41500	10412	42241
50	3500	60704	41500	6887	27572

Appendix 2

Illustration of benefit payment for females[3]

Age	Product A			Product B	
	Annuity Income	TPD/Death (X%)	TPD/Death (Y%)	Annuity Income	TPD/Death
30	3500	61091	41500	14762	53918
40	3500	61091	41500	10412	42265
50	3500	60704	41500	6887	27523

3 A premium charge of RM50,000 was used in the analysis are for both products, where Product A requires payment on a single premium basis and Product B requires an annual premium of RM5,000 for 10 years. The TPD or death benefit shown is an average annual payment. Data extracted from Sales Illustration obtained from both companies.

Appendix 3

R Code for Lee–Carter Mortality Projection

```
# extract mortality data from Excel file Msia_Deathrate_Male.csv
x <- scan(file="Msia_Deathrate_Male.csv", what = "character", skip = 1, sep = ',')
x.mat <- matrix(x, ncol = 18, byrow = TRUE)
mx <- x.mat[1:76, 2:18]
mx <- matrix(as.numeric(mx), nrow = 76, ncol = 17)
Age <- c(seq(length=76, from=25, to=100))
Year <- 2000:2016
dimnames(mx) = list(Age, Year)
n <- nrow(mx) # number of ages
m <- ncol(mx) # number of years
# Transpose data and get deaths and logrates
mx <- t(mx)
logrates <- log(mx)
# Do SVD
ax <- apply(logrates,2,mean) # ax is mean of logrates by column
clogrates <- sweep(logrates,2,ax) # central log rates (with ax subtracted) (dimensions
m*n)
svd.mx <- svd(clogrates)
# Extract first principal component
sumv <- sum(svd.mx$v[,1])
bx <- svd.mx$v[,1]/sumv
kt <- svd.mx$d[1] * svd.mx$u[,1] * sumv
# Forecasting kt - Fit kt using ARIMA model (0,1,0)
library(forecast)
fitkt <- Arima(kt,order=c(0,1,0),include.drift=TRUE)
kt.drift <- fitkt$coef
fitmx <- function (kt,ax,bx)
# Derives mortality rates from kt mortality index, following the Lee-Carter method
{clogratesfit <- outer(kt, bx) logratesfit <- sweep(clogratesfit,2,ax,"+") logratesfit}
logfit <- fitmx(kt,ax,bx)
dimnames(logfit)=list(Year, Age)
exp(logfit)
# Calculate the fitted value squared error
mxfiterror <- (exp(logfit)-mx)^2
# Project kt year 2017:2088
```

Appendix 3 *(Continued)*

```
y <- 1:72
kt.forecast <- kt[17] + (y*kt.drift)
Year.forecast <- 2017:2088
logfit.forecast <- fitmx(kt.forecast,ax,bx)
dimnames(logfit.forecast)=list(Year.forecast, Age)
logfit.forecast
# Combine actual and forecast logfit
all.logfit <- rbind(logfit,logfit.forecast)
fitrate <- exp(all.logfit)
write.csv(data.frame(fitrate),file="output_fitrate_mortmale.csv")
```